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COST MINIMIZATION OF POST-TENSIONED CONCRETE BEAMS USING EUROCODE 2-EC2

MINIMISATION DES COÛTS DES POUTRES EN BÉTON PRÉCONTRAINT PAR POST-TENSION SELON EUROCODE 2-EC2

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Abstract - In this paper, a model to calculate the optimum cost design of post-tensioned ordinary and high strength concrete T-beams is presented. The objective function comprises the costs of prestressed concrete, prestressing force and formwork. The constraint functions are set to meet the design requirements of Eurocode 2 (EC2). Several limit states are considered, including permissible tensile and compressive stresses at both transfer and service stages, prestressing force, ultimate flexural strength, ultimate shear strength and deflection limit, as well as the general rules of structural fire design. The cost optimization is developed through the use of the Generalized Reduced Gradient (GRG). Two examples have been included in order to illustrate the applicability of the proposed approach and solution methodology. The optimized results are compared to traditional design solutions to evaluate the performance of the developed cost model. It is shown; among others that, optimal solutions achieved using the present model can lead to substantial savings in the amount of construction materials to be used. In addition, the proposed approach is practically simple, reliable and computationally effective compared to classical designs procedures used by designers and engineers.

Keywords : Cost minimization design; Nonlinear optimization; Post-tensioned concrete beams; Eurocode2 (EC2); Fire resistance; Solver.

Résumé - Cet article présente la conception à coût minimal des poutres en T précontraintes par post-tension en béton ordinaire et en béton à haute résistance selon Eurocode 2. La fonction objective comprend le coût du béton, le coût de la force de précontrainte et le coût du coffrage. Les fonctions de contrainte sont définies pour satisfaire les exigences de conception de l'Eurocode 2 (EC2). Plusieurs états limites sont pris en compte, y compris les contraintes de traction et de compression admissibles en phases provisoires et de service, la force de précontrainte, la résistance ultime à la flexion, la résistance ultime au cisaillement et la flèche limite, ainsi que les règles générales de conception structurelle du feu. L'optimisation des coûts est développée grâce à l'utilisation du Gradient Réduit Généralisé (GRG). Deux exemples ont été inclus afin d'illustrer l'applicabilité de l'approche proposée et de la méthodologie développée. Les résultats optimisés sont comparés aux solutions de conception traditionnelles pour évaluer les performances du modèle de coût développé. Il est montré entre autres que les solutions optimales obtenues, en utilisant le modèle proposé, peuvent entraîner des économies substantielles dans les quantités de matériaux de construction mis en œuvre. De plus, l'approche proposée est pratiquement simple, fiable et efficace sur le plan des calculs par rapport aux procédures de conception classiques utilisées par les concepteurs et les ingénieurs.

Mots-clés : Conception à coût minimal; Optimisation non-linéaire; Poutres en béton par post-tension; Eurocode 2 (EC2); Résistance au feu ; Solveur.

1. Introduction

Structural designers have traditionally the task to develop designs that provide safety. Structural optimization on the other hand deals with the design of structural elements and systems employed in several engineering fields. One of the most common structural design methods involves decision making based on experience and intuition. The design of the structures both buildings and bridges is often mostly governed by cost rather than by weight considerations. A better design is achieved if an appropriate cost or objective function can be reduced. Safe and economical design of concrete structures can be achieved more by deciding on proper choices for the construction site, a practical overall layout of the structure and its resistant system, careful attention to construction detailing and sound construction practice.

Using numerical optimization as a design tool has several advantages: optimization techniques can greatly reduce the design time and yield improved, efficient and economical designs [1,2,3].

Structural elements with T-shaped sections are frequently used in industrial construction. They are used for repeated and large structures, because they are cost effective when using optimum cost design model which is of great value to practicing designers and engineers. One of the great advantages of T-beams sections is the economy of the amount of steel needed for reinforcement. Savings in material weight are very important due to elevation and transportation costs. The use of prestressing reduces significantly the weight of the structures since it reduces the amount of concrete. The prestressed post-tensioned T-beam system are frequently used in engineering practice and typically used in public works construction. In most construction sites, the concrete is post-tensioned. Post-tensioning is also used in segmental construction of large-span bridge girders. They are widely used for short span 20 m to medium span 40 m due to its moderate self-weight, structural efficiency, ease of fabrication, fast construction low maintenance, durability and aesthetic. A large number of papers have been published in recent years for optimal design of one beam simply supported pre-or-post-tensioned fully or partially prestressed normal-strength concrete (NSC) or high strength concrete (HSC). Barakat and al. [4] developed a general approach to the single objective reliability-based optimum (SORBO) design of prestressed beams (PCB) and solved by integrating PCB design and reliability analysis computer programs with an automated design optimization package using the feasible direction method. The proposed SORBO formulation provides a confidence range of the reliability-based optimum designs which lead to cost effective products. Sirca and Adeli [5] presented the total cost optimization of precast prestressed concrete I-beam bridge systems. The problem is formulated as a mixed integer-discrete nonlinear programming problem and solved using the robust neural dynamics model. The excellent convergence of the solution and the robustness of the proposed model are the main advantages and this model may be easily expanded to include other prestressed concrete beams types. Aydin and Ayvaz [6] optimized the cost of a prestressed concrete I-beam bridge superstructure by using a genetic algorithm. It is concluded that GA software can be used to efficiently optimize the shape and the topology of prestressed concrete bridge girders. Hermandez et al. [7] wrote software in order to produce the optimum layout of prestressing tendons in concrete beams. The software VTOP contains the necessary capabilities for daily applications and possesses, a user friendly graphical interface which helps in introducing new design techniques. Ashan et al. [8] presented an optimization approach to the design of simply supported post-tensioned prestressed concrete I-girder bridges. The proposed model is applied to a real life bridge project and shows considerable savings in cost. Rana et al. [9] implemented an evolutionary operation to the minimum cost design of continuous prestressed concrete bridge structure. Minimum design achieved by application of the proposed optimization approach to a practical design example leads to substantial savings in cost. Quanrata and al. [10] investigated the application of the constrained differential evolution algorithm (IDEA) for the optimum design of prestressed concrete beams. The ICDE algorithm implements a reliable mechanism to find the feasible region promptly and exhibits improved performance and stability when compared to genetic algorithm.

Concrete subjected to elevated temperatures in fire tests presents a decrease in compressive strength, tensile strength, elastic modulus, therefore an increase in peak strain and change in stress-strain relationship. It is important to know the behavior of high strength concrete beam exposed to fire as an accidental loading and a structural fire design analysis should be taken into account in accordance with design code [11, 12, 13].

Advances in numerical optimization methods, computer based numerical tools for analysis and design of structures and availability of powerful computing hardware have significantly helped the design process to ascertain the optimum design namely: [14, 15, 16, 17, 18, 19].

In this paper, a model to calculate the optimum cost design of post-tensioned ordinary and high strength concrete T-beams is presented. The objective function comprises the costs of prestressed concrete, prestressing force and formwork. The constraint functions are set to meet the design requirements of Eurocode 2 (EC2). Several limit states are considered, including permissible tensile and compressive stresses at both transfer and service stages, prestressing force, ultimate flexural strength, ultimate shear strength and deflection limit, as well as the general rules of structural fire design. The cost optimization is developed through the use of the Generalized Reduced Gradient (GRG). Two examples have been included in order to illustrate the applicability of the proposed approach and solution methodology. The optimized results are compared to traditional design solutions to evaluate the performance of the developed cost model. It is shown; among others that, optimal solutions achieved using the present model can lead to substantial savings in the amount of construction materials to be used. In addition, the proposed approach is practically simple, reliable and computationally effective compared to classical designs procedures used by designers and engineers.

2. Prestressed concrete optimization

Ultimate limit states (ULS) and serviceability limit states (SLS) for the optimum cost design of prestressed concrete T-beams are presented in the present study in accordance with the current European design code EC2 [20, 21, 22].

Considering the total cost minimization of simply supported prestressed concrete T-beams having the cross section shown in Figure 1.

The cross-sectional properties of the choose T-section are taken as follows:

$\theta = b_w/b$; $\beta = h_f/h$

Area of cross section: $A = bh[\theta + \beta(1-\theta)]$

Distance to the top fiber from the centroidal axis:

$y_t = v = h - y_b = h - v' = h/2[(\theta + \beta^2(1-\theta))/(\theta + \beta(1-\theta))]$

Distance to the bottom fiber from the centroidal axis:

$y_b = v' = [h[(\theta/2 + \beta(1-\theta)(1-\beta/2)]/(\theta + \beta(1-\theta))]$

Moment of inertia:

$I_c = bh^3/12[\theta + (1-\theta)\beta^3 + \theta(0.5 - y_b/h)^2 + \beta(1-\theta)(1-0.5\beta - y_b/h)^2]$

Section modulus to extreme bottom fiber:

$Z_b = I_c/y_b = I/v'$

Section modulus to extreme top fiber:

$Z_t = I_c/(h-y_b) = I/y_t = I/v$.

2.1-Definition of design variables

The design variables selected in this work for the optimization are listed in Table 1.

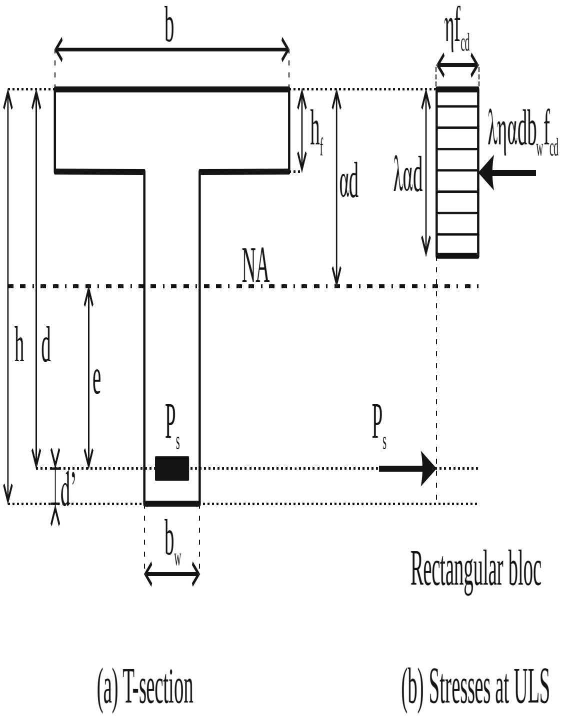

Figure 1: (a) Section en T entièrement précontrainte; (b) distribution des contraintes à l'état limite ultime (ELU)

Figure 1 : (a) Fully prestressed T- section; (b) Stresses at ultimate limit state (ULS)

Table 1: Definition of design variables

Tableau 1: Définition des variables de conception

Design variables	Defined variables
b	effective width of compressive flange
b_w	web width
h	total depth
d	distance from extreme compressive fiber to centroid of the prestressing force
h_f	flange depth
α	relative depth of compressive concrete zone
P_s	prestressing force
e	eccentricity of prestressing force

2.2-Objective function

The objective function to be minimized in the optimization problem is the total cost of construction material per unit length of the beam. This function can be defined as:

$$C_0 = C_c(b_w h + (b - b_w)h_f) + C_P P_S + C_f(b + 2h) \rightarrow \text{Min.} \quad (1)$$

Where:
C_0: Total cost per unit length of prestressed T-beam

C_P: Unit cost of prestressing force

C_c: Unit cost of prestressed concrete

C_f: Unit cost of formwork

It should be noted that in a cost optimization problem, the optimal values of the design variables are only affected by the relative cost values of the objective function and not by the absolute cost values. In other words, the absolute cost values affect the final value of the objective function but not the optimal values of the design variables.

The absolute cost C_0 can then be recovered from the optimized relative cost C by using the relation:

$$C_0 = C_c L C \quad (2)$$

Thus, the objective function to be minimized can be written as follows:

$$C = b_w h + (b - b_w)h_f + (C_P/C_c)P_s + (C_f/C_c)(b + 2h) \rightarrow \text{Min.} \quad (3)$$

The values of the cost ratios C_P/C_c and C_f/C_c vary from one country to another and eventually from one region to another within the same country.
The values of these cost ratios can be estimated on the basis of data provided in applicable unit price books of construction materials [23, 24].

2.3 -Formulation of the design constraints

The following constraints for the post-tensioned T-beams are defined in accordance with the design code specifications of the EC2:

a- Ultimate limit states:
Ultimate flexural strength constraints**:**

$$M_{Ed} \leq f_{cd}(b - b_w)h_f(d - 0.5h_f) + \lambda\eta b_w d^2 f_{cd}\alpha(1 - 0.5\lambda\alpha) \quad (4)$$

(External moment ≤ Resisting moment of the cross section)

$$f_{cd}(b - b_w)h_f + \lambda\eta b_w \alpha d f_{cd} - P_s = 0 \quad (5)$$

(Internal force equilibrium)

$$1 \leq \frac{\alpha d}{h_f} \quad (6)$$

(T-section behaviour)
Ultimate shear strength constraint:

$$V_{Ed} \leq V_{Rd,max} = \alpha c_w \nu_1 f_{cd} b_w z/(tg(\theta) + cotg(\theta)) \quad (7)$$

b- Serviceability limits states:

Flexural working stress constraints at transfer:

$$-\frac{P_s}{A\kappa} - \frac{P_s e}{Z_t\kappa} - \frac{M_G}{Z_t} \leq f_{tt} \quad \text{at top face} \quad (8)$$

$$-P_s/A\kappa + P_s e/Z_b\kappa + M_G/Z_b \geq f_{tc} \quad \text{at bottom face} \quad (9)$$

Flexural working stress constraints at service:

$$-P_s/A - P_s e/Z_t - M_{sls}/Z_t \geq f_{sc} \quad \text{at top face} \quad (10)$$

$$-P_s/A + P_s e/Z_b + M_{sls}/Z_b \leq f_{st} \quad \text{at bottom face} \quad (11)$$

Deflection due to both dead and live loads:

$$\frac{5L^2(-P_s e + M_{sls})}{48E_c I_c} \leq \delta_{lim} \quad (12)$$

c- Prestressing force:

$$P_{s\,min} \leq P_s \leq P_{s\,max} \quad (13)$$

d- Constraint for eccentricity of the prestressing force:

$$0 \leq e \leq (y_b - d') \quad (14)$$

e- Constraints for minimum section modulus:

$$Z_t \geq (M_{sls} - \kappa M_G)/(\kappa f_{tt} - f_{sc}) \quad (15)$$

$$Z_b \geq (M_{sls} - \kappa M_G)/(f_{st} - \kappa f_{tc}) \quad (16)$$

f- Geometric performance of T-Section:

$$0.30 \leq I_c/A y_b y_t \leq 0.50 \quad (17)$$

i- Design constraints:

$$L/25 \leq h \leq L/15 \quad (18)$$

$$b/h_f \leq 8 \quad (19)$$

$$(b - b_w)/2 \leq L/10 \quad (20)$$

$y_b + y_t - h = 0$ (21)

$d = y_t + e$ (22)

$h_{fmin} \leq h_f$ (23)

$b_{wmin} \leq b_w$ (24)

$b/b_w \geq 3$ (25)

j- Non-negativity variables:

$b, b_w, h, d, h_f, P_s, \alpha > 0$ (26)

The cross-sectional properties (A, y_b, y_t, Z_b, Z_t, I_c) of the chosen T-section are:

$\theta = b_w/b$; $\beta = h_f/h$

Area of cross section: $A = bh[\theta + \beta(1-\theta)]$

Distance to the top fiber from the centroidal axis:

$y_t = \nu = h - y_b = h - \nu' = h/2[(\theta + \beta^2(1-\theta))/(\theta + \beta(1-\theta))]$

Distance to the bottom fiber from the centroidal axis:

$y_b = \nu' = [h[(\theta/2 + \beta(1-\theta)(1-\beta/2)]/(\theta + \beta(1-\theta))]$

Moment of inertia: $I_c = bh^3/12[\theta + (1-\theta)\beta^3 + \theta(0.5 - y_b/h)^2 + \beta(1-\theta)(1-0.5\beta - y_b/h)^2]$

Section modulus to extreme bottom fiber: $Z_b = I_c/y_b = I/\nu'$

Section modulus to extreme top fiber: $Z_t = I_c/(h-y_b) = I/y_t = I/\nu$.

2.4-Formulation of the optimum cost design problem

The optimum cost design of prestressed T-beams can be stated as follows: For given material properties, loading data and constant parameters, find the design variables defined in Table (1) that minimize the cost function defined in Eq. (3) subjected to the design constraints given in Eq.(4) through Eq.(26).

2.5-Solution methodology: generalized reduced gradient method

The objective function Eq.(3) and the constraints equations, Eq.(4) through Eq.(26), form together a nonlinear optimization problem. The reasons for the nonlinearity of this optimization problem are essentially due to the expressions for the cross sectional area, bending moment capacity and other constraints equations. Both the objective function and the constraint functions are nonlinear in terms of the design variables. In order to solve this nonlinear optimization problem, the solution process is performed by using the GRG nonlinear method. With the GRG algorithm, it has been found that it is a powerful solver for many design problems. The reduced gradient method is based on a simple variable elimination technique for equality constrained problems. Dependent and independent variables are identified in the linearized sub-problem and the dependent variables are eliminated from it. The generalized reduced gradient method is an extension of the reduced gradient method to accommodate nonlinear inequality constraints. The GRG algorithm available in the MS Excel Solver Module is used to solve the nonlinear optimization problem.

For the purpose of implementing the GRG method, Solver add-in of Microsoft Excel was used as an optimization tool. In order to use the Excel Solver, two main steps need to be followed: k) preparation of an Excel worksheet for the problem, identifying the cells allocated for the design, objective function and the constraints, kk) the solver is then invoked, which results in the display of the Solver Parameters dialogue box. In this box, the actual problem that has to be solved is defined. The cells that contain the variables, objective function and the cells defining different constraints for the problem are identified.

The main steps in GRG algorithm can be summarized as follow:

Step 1: Find a feasible solution and divide it into basic and non-basic subsets

Step 2 : Find the search direction

Step 3 : Perform a line search

Step 4 : Check the feasibility

Step 5 : Change the basis

If the variable approaches to its boundary limits, the variable must leave out the basic variable set and become a nonlinear variable. Another non-basic variable with the largest absolute value of gradient is selected to enter the basic variable set. The corresponding basic and non-basic matrices must also be set to be changed. Then go the Step 2.The interested reader is directed to [25, 26] for further details.

3. Numerical results

A typical example problem is now considered, followed by a comparison between the standard design solution and the optimal solution obtained.

3.1- Design example A for prestressed normal-strength concrete (NSC) T-beams

As previously mentioned, the design constraints are defined in accordance with the code design specifications of EC2. The optimal solutions are compared to the standard design solutions obtained in accordance with EC2 design code.

The study of normal strength concrete prestressed T-beam corresponds to a T-beam simply supported at its ends and pre-designed in accordance with provisions of EC2 design European code. The T-beam supports, in addition to its self-weight, superimposed dead load and live load. It is made from C40/50 concrete. The concrete grade at the time of stressing can be taken as C30/37. The beam span is L=30 m. The T-beam is post-tensioned by the prestressing force P_s which is assumed constant on every section with zero eccentricity at each end and **e** required at mid-span.

The corresponding pre-assigned parameters are defined in Table 2 below:

Table 2: Material properties, loading and cost data and other parameters for the NSC T-beam example

Tableau 2 : Propriétés des matériaux, données de chargement et de coût et autres paramètres pour l'exemple d’une poutre en T en béton à résistance normale (BRN)

Data for loads and concrete dimensions	**Data for Normal-Strength Concrete (NSC)**	**Data for Prestressing force and costs**
L = 30.00m	*At transfer:*	*Prestressing*
δ_{lim} =L/250 =0.12m	Grade: C30/37	*force:*
h_{fmin}=0.15m	f_{ck} = 30MPa; γ_c	P_t=k_1P_{jack}
b_{wmin}=0.30m	= 1.5	P_s=k_2P_{jack}
h_{min}=L/25 =1.20m;	f_{cd}= 20MPa	k_1=0.90
h_{max}=L/15 =2.00m	f_{tt}=f_{ctm}=2.9MPa	k_2=0.75
d’=0.10m	f_{tc}= -18.0MPa	$P_{s\ min}$ = 2MN
M_G =5MNm	λ=0.800; η=1.00	$P_{s\ max}$ = 10MN
M_Q=2MNm	*At service:*	*Relative cost*
M_{SLS}=7MNm	Grade: C40/50	*ratio of*
M_{Ed}=9.75MNm	f_{ck} = 40MPa	*materials:*
V_{Ed}=1.5 MN	f_{cd}= 26.67MPa	C_p/C_c = 20
v_1=0.6; α_{cw}=1.25	f_{st}=f_{ctm}=3.5MPa	C_f/C_c = 0.01
	f_{sc}=-0.6x40= -	
	24.0MPa	
	λ=0.800; η=1.00	
	E_{cm}=35000MPa	

3.2-Comparison between the optimal cost design solution and the standard design approach

The vector of design variables as obtained from the conventional design solution, the optimal cost design solution using the proposed approach are shown in Table 3 below.

Table 3: Comparison between the classical solution and the optimal solution for NSC

Tableau 3 : Comparaison entre la solution classique et la solution optimale pour béton à résistance normale (BRN).

Vector solution for C_P/C_c=20 C_f/C_c=0.01	Classical solution for NSC	Optimal Solution for NSC
b[m]	1.30	0.97
b_w[m]	0.50	0.30
h_f[m]	0.15	0.15
h[m]	1.94	2.00
d[m]	1.84	1.90
α	0.20	0.29
P_s[MN]	7.10	6.20
e[m]	0.95	1.03
C	143.142	124.750
Gain		15%

From the above results, it is clearly observed using the values of the relative costs C_P/C_c=20, C_f/C_c=0.01 and comparing the classical with the optimal solution, that a significant gain equal to 15% can be obtained using the proposed design formulation.

3.3- Cost sensitivity analysis for example a: t-beam with normal strength concrete (NSC)

3.3.1-Variation of relative gain in percentage (%) versus unit cost ratio C_p/C_c for a given cost ratio C_f/C_c

The relative gains can be determined for various values of the unit cost ratios C_p/C_c for two given unit cost C_f/C_c ratios. The corresponding results are reported in Table 4 and illustrated graphically in Figure 2 for C_f/C_c=0.01.

Table 4 : Variation of relative gain in percentage (%) versus unit cost ratio C_p/C_c of construction materials for a given value C_f/C_c=0.01

Tableau 4 : Variation du gain relatif en pourcentage par rapport au rapport de coût unitaire C_p/C_c des matériaux de construction pour une valeur donnée C_f/C_c = 0,01

C_P/C_c	Gain % For C_f/C_c=0.01
1	18,5
2	16,58
3	15,89
4	15,53
5	15,31
6	15,17
7	15,06
8	14,98
9	14,92
10	14,87
20	14,74
30	14,57
35	14,55
50	14,51

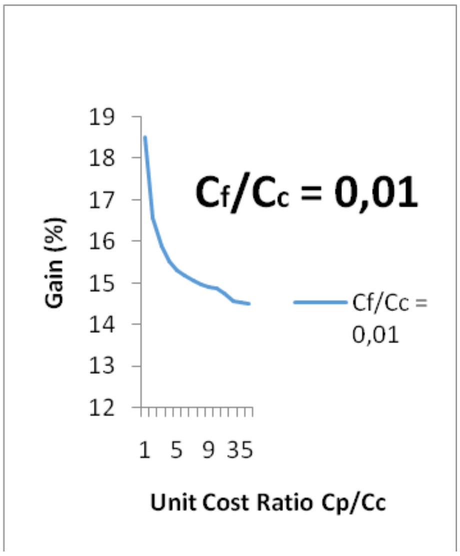

Figure 2: Variation of relative gain in percentage for a given cost ratio C_f/C_c = 0,01 for different values of C_p/C_c

Figure 2 : Variation du gain relatif en pourcentage pour un rapport de coût donné C_f/C_c = 0,01 pour différentes valeurs de C_p/C_c

From Table 4 and Figure 2, the gain decreases with the increase of the unit cost ratio C_p/C_c.

3.3.2-Variation of relative gain in percentage (%) versus unit cost ratio c_f/c_c for a given cost ratio C_p/C_c:

The relative gains can be determined for various values of the unit cost ratios C_f/C_c for two given unit cost C_p/C_c ratios. The corresponding results are reported in Table 5.

Table 5 : Variation of relative gain in percentage versus unit cost ratio C_f/C_c of construction materials for a given value C_p/C_c=20

Tableau 5. Variation du gain relatif en pourcentage par rapport au rapport de coût unitaire C_f/C_c des matériaux de construction pour une valeur donnée C_p /C_c = 20

C_f/C_c	Gain % For C_p/C_c=20
0.01	14.65
0.02	14.64
0.03	14.63
0.04	14.63
0.05	14.61
0.10	14.45
1	14.20

From Table 5, the gain decreases with the increase of the unit cost ratio C_f/C_c.

3.4 -Design example b for high strength concrete (HSC) T-beams:

As previously mentioned, the design constraints are defined in accordance with the code design specifications of EC2. The optimal solutions are compared to the standard design solutions obtained in accordance with EC2 design code.

The study of high strength concrete prestressed T-beam corresponds to a T-beam simply supported at its ends and pre-designed in accordance with provisions of EC2 European design code. The T-beam supports, in addition to its self-weight, superimposed dead load and live load. It is made from C70/85 concrete. The concrete grade at the time of stressing can be taken as C60/75. The beam spans is L=40 m. The T-beam is post-tensioned by the prestressing force P_s which is assumed constant on every section with zero eccentricity at each end and **e** required at mid-span.

The corresponding pre-assigned parameters are defined as in Table 6 below:

Table 6: Material properties, loading and Cost data and other parameters for the HSC T-beam example.

Tableau 6 : Propriétés des matériaux, données de chargement et de coût et autres paramètres pour l'exemple d'une poutre en T en béton à haute résistance (BHR).

Data for loads and concrete dimensions	**Data for High Strength Concrete- HSC**	**Data for Prestressing force and costs**
L = 40.00m δ_{lim} = L/300 =0.13m h_{fmin}=0.10m b_{wmin}=0.25 m h_{min}=L/25 =1.60m; h_{max}=L/15 =2.67m d'=0.10m M_G =6MNm M_Q=3MNm M_{SLS}=9MN m M_{Ed} = 12.6MNm V_{Ed}= 2.9MN ν_1=0.6; α_{cw}=1.25	*At transfer:* Grade: C60/75 f_{ck} = 60MPa; γ_c = 1.5 f_{cd}= 40MPa f_{tt}=f_{ctm}=4.4MPa f_{tc}=-36MPa λ=0.775 ; η=0.975 E_{cm}= 39000MPa *At service:* Grade: C70/85 f_{ck} = 70MPa f_{cd}= 46.67MPa f_{st}=f_{ctm}=4.6MPa f_{sc}=-0.6x40=-42MPa λ=0.750 ; η=0.900 E_{cm}= 41000MPa	*Prestressing force:* P_t=k_1P_{jack} P_s=k_2P_{jack} k_1=0.90 k_2=0.75 $P_{s\,min}$ = 2MN $P_{s\,max}$ = 10MN *Relative cost ratio of materials:* C_p/C_c = 10 C_f/C_c = 0.01

3.5- Comparison between the optimal cost design solution and the standard design approach:

The vector of design variables as obtained from the optimal cost design solution using the proposed approach is shown in Table 7 below.

From the above results, it is clearly observed using the values of the relative costs

C_P/C_c=10, C_f/C_c=0.01 and comparing the classical with the optimal solution, that a significant gain equal to 10% can be obtained using the proposed design formulation.

3.6- Fire design for t-beams (at elevated temperature θ = 350°c)

In addition to the reducing of the HSC member's size s comparing to normal-strength concrete (NSC), the risk of spalling is higher in HSC for the following reasons:

i)low permeability of HSC retains the moisture inside the concrete, resulting in a high moisture content being for prolonged periods; ii) low porosity of HSC creates higher pore pressure; iii) HSC tends to be subject to higher compressive stresses than lower-strength concrete.

Static equilibrium of flexural moments should be ensured under design fire conditions. The moment of resistance of the section should be calculated for fire conditions in accordance with EC2. The design values of material properties at elevated temperature for T-beams exposed to fire are used. The combination rules for mechanical actions at fire conditions are respected. The Tabulated method is used for determining the minimum dimensions and cover in design procedure of HSC beams at elevated temperature. The minimum web width b_w should be increased for beams exposed on one side only by 0.3a; where a, is the axis-distance for Class C70/85. However, the axis-distance, a, should be increased by the factor 1.3 for Class C70/85 of HSC-Tabulated Data. When using Tabulated Data, no further checks are required concerning shear capacity.

List of design constraints:

a-Ultimate flexural strength constraints**:**

$$M_{Ed} \leq f_{cd,\theta}(b-b_w)h_f(d-0.5h_f) +$$

$$\lambda\eta b_w d^2\, f_{cd,\theta}\alpha(1-0.5\lambda\alpha) \quad (27)$$

Table 7: Optimal solution for HSC at ambient temperature ϴ=20°C

Tableau 7 : Solution optimale pour BHR à température ambiante ϴ = 20 °C

Vector solution for C_P/C_c=10 C_f/C_c=0.01	Classical Solution for HSC	Optimal solution for HSC at ambient temperature ϴ=20°C
b[m]	1.10	1.04
b_w[m]	0.35	0.30
h_f[m]	0.13	0.13
h[m]	2.70	2.67
d[m]	2.43	2.57
α	0.06	0.05
P_s[MN]	6.25	5.68
e[m]	1.30	1.37
C	63.607	57.798
Gain		10%

(External moment ≤ Resisting moment of the cross section)

b- Internal equilibrium force:

$$f_{cd,\theta}(b-b_w)h_f+\lambda\eta b_w\alpha d\, f_{cd,\theta}-P_s=0 \quad (28)$$

c- Cover constraint

$$h--d \geq 1.3a-\frac{\phi_{Tendon}}{2} \quad (29)$$

$$h_{fmin,\theta} \leq h_f \quad (30)$$

$$b_{wmin,\theta} \leq \quad (31)$$

d- T-section behaviour: Eq. (6)

e- Prestressing force: Eq. (13)

f- Constraint for eccentricity of the prestressing force: Eq. (14)

i- Geometric performance of T-Section: Eq. (17)

j- Design constraints: Eq. (18), Eq. (19),

Eq.(20), Eq.(21), Eq.(22), Eq.(25).

k- Non-negativity variables: Eq. (26)

3.7- Numerical examples

The corresponding pre-assigned parameters are defined in Table 8 below:

Table 8 : Loading and input data for both NSC and HSC T-beam

Tableau 8 : Chargement et données d'entrée pour poutre en T en BRN et BHR

Input data for fire design by using Tabulated Data at θ=350°C	Input data for NSC characteristics	Input data for HSC characteristics:
Standard fire exposure: 60 minutes (R60) *Minimum web thickness b_w for R60:* $b_{wmin,\Theta}$=100mm *Minimum axis-distance*: a: a=55mm h-d=cover c c=63.65mm	*Class NSC*: C40/50 f_{ck} = 40MPa $f_{c,\theta}=0.75f_{ck}$ γ_c =1.3 $f_{cd,\theta}$= 23.08MPa $h_{fmin,\Theta}$=0.10m $b_{wmin,\Theta}$=0.15m *Load combination:* G+0.5Q M_{Ed}= 6 MNm (NSC)	*Class HSC*: C70/85 f_{ck} = 70MPa $f_{c,\theta}=0.75f_{ck}$ γ_c =1.3 $f_{cd,\theta}$= 40.38MPa; $h_{fmin,\Theta}$=0.10m $b_{wmin,\Theta}$=0.10m *Load combination:* G+0.5Q M_{Ed}= 7.50MNm (HSC)

The vector of design variables as obtained from the fire design solution using the proposed approach is shown in Table 9 below.

Table 9 : Optimal design solution for fire design of HSC and NSC

Tableau 9 : Solution de conception optimale pour la résistance au feu de BHR et BRN

Vector solution	Optimal solution for NSC at elevated temperature ϴ=350°C C_P/C_c=20, C_f/C_c=0.01	Optimal solution for HSC at elevated temperature ϴ=350°C C_P/C_c=10, C_f/C_c=0.01
b[m]	1.32	0.88
b_w[m]	0.44	0.17
h_f[m]	0.16	0.11
h[m]	2.00	2.67
d[m]	1.36	2.03
α	0.12	0.07
P_s[MN]	4.66	3.8
e[m]	0.49	0.90

4. Conclusions

The following conclusions are drawn from this study:

01- The problem formulation of the optimal cost design of prestressed T-beams can be cast into a nonlinear programming problem.

02- Optimal values of the design variables are only affected by the relative cost values of the objective function and not by the absolute cost values.

03- The observations derived from the results of the optimal solutions reveal that the use of the optimization based on the optimum cost design concept may lead to substantial savings in the amount of the construction materials to be used in comparison to classical design solutions of prestressed T-beams. A significant cost saving of the order of 15% for NSC and 10% for HSC was achieved compared to conventional solutions.

04- The objective function and the constraints considered in this paper are illustrative in nature. This approach based on nonlinear mathematical programming can be easily extended to other sections commonly used in structural design. More sophisticated objectives and considerations can be readily accommodated by suitable modifications of the present optimal cost design model.

05- The gain decreases with the increase of both the unit cost ratio C_p/C_c and C_f/C_c ratio.

06- The proposed methodology for optimum cost design is effective and more economical in regards to the classical methods. The results of the analysis show that the optimization process presented herein is effective and its application appears feasible.

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Notations

The following symbols are used in this paper:

a Axis-distance
AN Neutral axis
b_{wmin} Minimum width of web
C Total relative cost of prestressed T-beam
C Cover
C_0 Total cost of T-beam
C30/37 Class of ordinary concrete
C40/50 Class of ordinary concrete
C60/75 Class of HSC
C70/85 Class of HSC
C_c Unit cost of concrete
C_f Unit cost of formwork
C_p Unit cost of prestressing force
d' Distance from bottom fiber to centroid of the prestressing force
E_{cm} Young's modulus for concrete
E_{cm} Modulus of elasticity of concrete

f_{cd} Design value of concrete compressive strength

f_{ck} Characteristic compressive cylinder strength of concrete at 28 days

f_{ctm} Tensile strength of concrete

f_{sc} Permissible stress at service in compression

f_{st} Permissible stress at service in tension

f_{tc} Permissible stress at transfer in compression

f_{tt} Permissible stress at transfer in tension

G Dead loads
h_{fmin} Minimum depth of flange
h_{max} Maximum total depth
h_{min} Minimum total depth
I_c Moment of inertia
L Beam span
M_{Ed} Maximum positive moment at ULS
M_G Maximum design moments under dead loads
M_Q Maximum design moments under live loads
$M_{Rd,max}$ Maximum resistant moment
M_{SLS} Maximum positive moment at SLS
n Ratio of the modulus of elasticity of steel to that of concrete (n=9 for HSC)
P_{jack} Force at jacking, $P_{jack} = P_s/0.75$
P_s Total prestress at service
$P_{s\ max}$ Maximum prestressing force at service
$P_{s\ min}$ Minimum prestressing force at service
P_t Initial prestressing force
S500 Grade of steel
f_{ck} Characteristic compressive cylinder strength of ordinary or HSC at 28 days
f_{cd} Design value of concrete compressive strength
γ_c Partial safety factor for concrete
η Design strength factor
λ Compressive zone depth factor
V_{Ed} Maximum shear at ULS
V_G Maximum design shears under dead loads
V_Q Maximum design shears under live loads
$V_{Rd,max}$ Maximum resistant shear force
z Lever arm
α Relative depth of compressive concrete zone

M_G Maximum design moments under dead loads

M_Q Maximum design moments under live loads

d Effective depth
Q Live loads
α_{cw} Factor depending on prestressing force

γ_c Partial safety factor for concrete
δ Deflection
δ_{lim} Limit deflection
η Factor for design strength
θ Temperature
Θ Angle between concrete compression struts and the main chord

κ Ratio of the final prestressing force after all losses to the initial prestressing force $P_s = \eta P_t$
λ Compressive zone depth factor
ν_1 Adimensional coefficient
ϕ Diameter of tendon in T-beam
d' Cover