OPTIMIZATION OF RIGID PAVEMENTS IN ACCORDANCE WITH PCA DESIGN METHOD

OPTIMISATION DES CHAUSSÉES RIGIDES CONFORMÉMENT À LA MÉTHODE DE CONCEPTION PCA

Réception : 24/02/2019 **Acceptation : 16/05/2019** **Publication : 30/06/2019**

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Abstract

This work presents a method for optimizing the cost of rigid pavements according to PCA method. The objective function includes the costs of concrete, formwork and prepared subgrade soil support. All the constraints functions are set to meet design requirements of PCA and current practices rules. The optimization process is developed through the use of the Generalized Reduced Gradient algorithm. An example is considered in order to illustrate the applicability of the proposed design model and solution methodology. It is concluded that this approach is economically more effective comparing to conventional design methods used by designers and engineers.

Keywords : Optimization, Concrete pavement, PCA method, Fatigue Analysis, Erosion Analysis, Algorithm.

Résumé

Ce travail présente une méthode d'optimisation du coût des chaussées rigides selon la méthode PCA. La fonction objective inclut les coûts du béton, du coffrage et la plateforme préparée. Toutes les fonctions de contraintes sont définies pour répondre aux exigences de conception de la méthode PCA et aux règles de pratique courante. Le processus d'optimisation est développé à l'aide de l'algorithme du gradient réduit généralisé. Un exemple est considéré afin d'illustrer l'applicabilité du modèle de conception proposé et de la méthodologie de la solution. Nous concluons que cette approche est économiquement plus efficace que les méthodes de conception conventionnelles utilisées par les concepteurs et les ingénieurs.

Mots-clés : Optimisation, Chaussée en béton, Méthode PCA, Analyse de fatigue, Analyse d'érosion, Algorithme.

1. Introduction

Concrete pavement is often used as a superstructure element as road slab for highways, airports, industrial grounds, streets, parking areas. Rigid pavement with and without base course are used in many countries all around the world. It has a large number of advantages such as long life spans, negligible maintenance, user and environment friendly and lower cost if they are designed correctly and constructed well. Because of its rigidity and high tensile strength, a rigid pavement tends to distribute the load over a relatively wide area of sub-grade, and a major portion of the structural capacity is supplied by the slab itself. Concrete

pavement has good design period vs bituminous. It is generally better able to cope with unexpected loads and fuel spillages in industrial estates and service areas. Concrete road is generally able to maintain an adequate skid resistance under heavy traffic for longer than bituminous surfacing [1, 2, 3, 4].

The pavements were designed under different traffic conditions and different soil parameters.

Pavement design methods are generally grouped into two major types, namely purely empirical approach and mechanistic-empirical approach. The Portland Cement Association's thickness design procedure (PCA method) is the most well-known, widely-adopted, and mechanically based procedure for the thickness design of jointed concrete pavements. The PCA thickness design criteria are to limit the number of load repetitions based on both fatigue analysis and erosion analysis. Cumulative damage concept is used for the fatigue analysis to prevent the first crack initiation due to critical edge stresses, whereas the principal consideration of erosion analysis is to prevent pavement failures such as pumping, erosion of foundation, and joint faulting due to critical corner deflections during the design period. The design factors considered by the PCA method include the design period, the flexural strength of concrete (or the concrete modulusof rupture), the modulus of sub-base-subgrade reaction, design traffic (including load safety factor, axle load distribution), with or without doweled joints and a tied concrete shoulder. The presence of doweled joints will affect the erosion analysis while the presence of concrete shoulder will affect both fatigue and erosion analysis [5, 6, 7, 8,9,10,11,12,13, 14, 15].

Structural designers have traditionally the task to develop designs that provides safety. Structural optimization on the other hand deals with the design of structural elements and systems employed in several engineering fields. One of the most common structural design methods involves decisions making based experience and intuition. The design of the structures both buildings, bridges and roads are often governed mostly by cost rather than by weight considerations. A better design is achieved if an appropriate cost or objective function can be reduced. The structural performances depend on the optimization techniques. The numerical optimization is one of the tools that helps provide the desired results in a timely and economical fashion[16, 17, 18, 19]. The effectiveness of the optimization method depends on both the algorithm and the software in use. Many algorithms have been developed and evaluated for practical optimization. The use of an algorithm that provides reliable results for the problem of interest is important. The use of constrained minimization methods necessitates for the design variables to be modified successively during the design process by moving in the design space from one design point to another. Constrained optimization is a very active field of research and many algorithms have been developed. Nonlinear structural analysis is an example where optimization can be used to solve a nonlinear cost minimization problem. It was shown that the minimum cost optimum design of simple structural elements could be stated as a nonlinear mathematical programming problem in a design variables space. Using numerical optimization as a design tool has several advantages: optimization techniques can greatly reduce the design time and yield improved, efficient and economical designs. Advances in numerical optimization methods, computer based numerical tools for analysis and design of structures and availability of powerful computing hardware have significantly helped the design process to ascertain the optimum design namely. During the seventies a large number of design problems were solved using these optimization techniques. It is clear that there is a need to develop optimum design equations for rigid pavements. Pavement engineers have identified the importance of taking advantage of the available routines for the optimum design of pavement structures[20, 21, 22, 23, 24]

This work presents a method for optimizing the cost of rigid pavements according to PCA method. The objective function includes the costs of concrete, formwork and prepared subgrade soil support. All the constraints functions are set to meet design requirements of PCA and current practices rules. The optimization process is developed through the use of the Generalized Reduced Gradient algorithm. An example is considered in order to illustrate the applicability of the proposed design model and solution methodology. It is concluded that this approach is economically more effective comparing to conventional design methods used by designers and engineers.

2. PCA design method for rigid pavements

2.1 Design input parameters, assumptions and factors

The PCA method considers the following factors: i) concrete flexural strength or modulus of rupture (f'_{cf}); ii) modulus of subgrade reaction (k), iii) cumulative truck traffic during the design period, categorized by axle type and load; iv) load safety factor (LSF). The input parameters that influence the design of rigid pavements based on the design procedure of PCA method are: project details, traffic details, structural details and the design coderequirements. The typical cross-section of rigid pavement considered in this study is shown in Figure 1.

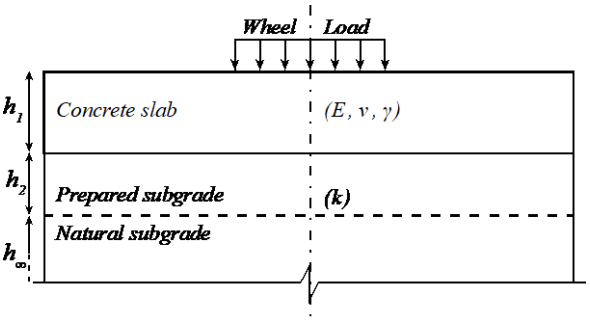

Figure 1 : Typical cross-section of rigid pavement.

Figure 1 : Coupe transversale typique d'une chaussée rigide

The material properties were assumed as below:

Design compressive strength of concrete: f'_c
Concrete modulus of rupture: $f'_{cf}=0,75\ (f'_c)^{0.5}$; the empirical relationship is commonly used to predict the modulus of rupture from the compressive strength (where f'_c and f'_{cf} are in MPa (SI units)).
Modulus of prepared subgrade reaction: k
Modulus of rigidity of concrete slab pavement: $D= Eh_1^3/12(1-\upsilon^2)$
Radius of relative stiffnessof the slab-sub-grade system: $L^4= D/k= Eh_1^3/12(1-v^2)k$, $L=(Eh_1^3/(12(1-v^2)k)^{0.25}$
Modulus of elasticity of concrete: E; Poisson's ratio of concrete: v; Unit weight of concrete: γ.

The axle load distribution of truck traffic is required to compute the expected number of axles of various weights during the design period. The axle load distributions for those four traffic categories were adopted for this study.
Load Safety Factor for axle loads: LSF
Average Design Traffic Trucks:
ADTT = 365(ADT)(T)(D)(LDF)(G)(N)
Pavement Design Life: N
Average Daily Traffic: ADT
Direction Distribution Factor: D
Lane Distribution Factor: LDF
Percent trucks: T
Annual traffic growth rate: r
Growth factor (PCA method): $G= (1+r)^{N/2}$

2.2 PCA Fatigue modelbased on tensile stress due to edge loads

Fatigue damage due to slab flexure and based on the edge stress because the edge stress on mainline pavements without concrete shoulders is much greater than on those with tied concrete shoulders. Figure 2 shows critical load positions.

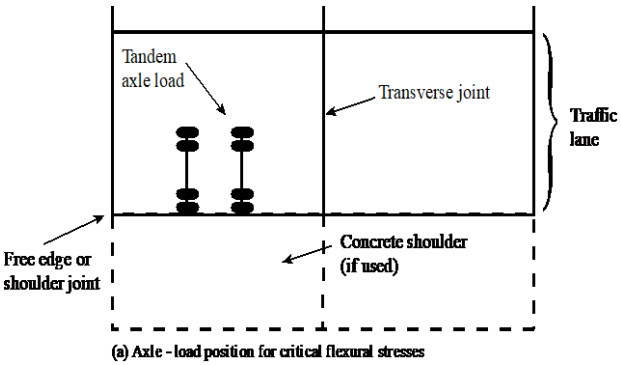

Figure 2: Critical loading position for fatigue analysis.
Figure 2 : Position de chargement critique pour l'analyse de la fatigue.

The PCA equations are as follows:
The equations for the fatigue of concrete are:

$$\begin{cases} \log(N_f) = \dfrac{0.9718 - S_r}{0.0828} \; when \;\; S_r > 0.55 \\ N_f = \left(\dfrac{4.2577}{S_r - 0.4325}\right)^{3.268} \; when \;\; 0.45 \le S_r \le 0.55 \\ N_f = Unlimited \;\; when \;\; S_r < 0.45 \end{cases} \quad (1)$$

Where:

$S_r= \sigma_{eq}/f'_{cf}$is the stress ratio factor

N_f is the allowable load repetitions based on fatigue; σ_{eq}, the equivalent stress (MPa); f'_{cf} and LSF are similar to the previous definitions;
Equivalent stress calculations:

Values of the equivalent stress (σ_{eq}) were obtained using the regression equations as follows:

$$\sigma_{eq} = \left(\frac{6M_e}{h_1{}^2}\right) f_1 f_2 f_3 f_4 \quad (2)$$

Where

h_1: thickness of the slab; fi: are additional adjustment factors; M_e: maximum edge moment

f_1: adjustment factor for the effect of axle loads and contact areas;f_2: adjustment factor for a slab with no concrete shoulder;f_3: adjustment factor to account for the effect of truck placement on the edge stress;f_4: adjustment factor to account for the increase in concrete strength with age after the 28 th day, along with a reduction in concrete strength by one coefficient of variation CV (PCA used CV=15%, f_4=0.953).

PCA regression equations for maximum edge moment:

Single Axle (SA)/ No Shoulder (NS):

M_e=-1600 + 2525 log(L) + 24.52L+0.204L^2 SA/NS

Tandem Axle (TA)/ No Shoulder (NS):

M_e= 3029-2966.8log(L) + 133.69 L-0.0632 L^2TA/NS

Single Axle (SA)/ With Shoulder (WS):

M_e=(-970.4 + 1202.6log(L) + (53.587L)(0.8742+0.01088$k^{0.447}$) SA/WS

Tandem Axle (TA)/ With Shoulder (WS):

M_e=(2005.4-1980.9log(L) +99.008L)(0.8742+0.01088$k^{0.447}$) TA/WS

L and k: same definitions as previously described.

Additional adjustment factors: f_i

f_1= $(24/SAL)^{0.06}(SAL/18)$ SA
SAL: Single Axle Load
f_1= $(48/TAL)^{0.06}(TAL/36)$ TA
TAL: Tandem Axle Load
f_2=0.892+h_1/85.71-h_1^2/3000 NS

f_2=1 WS

f_3= 0.894 for 6% truck at the slab edge

f_4=1/(1.235(1-CV); (PCA used CV =15%; f_4=0.953);

SAL and TAL: correctors factors for actual single axle load (SAL) or tandem axle load (TAL).

The combined effect of the four adjustment factors is to reduce the moment, M_e, calculated using the regression equations. This regression has a mean value between 0.80 and 0.85 for the No Shoulder (NS) cases ($0.80 \leq f_1 f_2 f_3 f_4 \leq 0.85$).

Miner's hypothesis for design calibration:
The simplicity of Miner's hypothesis of linear damage accumulation has resulted in the wide acceptance in most engineering fields, including rigid pavement design.
Cumulative Fatigue Damage :

$$D = \sum_{i=1}^{m} \left(\frac{n_i}{N_{if}}\right) \quad (3)$$

Where,
N_f: is the maximum allowable load repetitions given conditions (axle type (single, tandem, or tridem)) ; n_i : expected number of load repetitions at conditions (single, tandem, or tridem).

2.3 PCA Erosion model based on deflections due to corner loads

Erosion damage due to foundation compression. It occurs at the pavement corner and is affected by the type of joint. In the PCA model, erosion damage is related to power. The PCA defines power exerted by each axle pass at the slab corner as the product of corner deflection and pressure at the slab base-subbase

interface divided by the length of the deflection basin, which is a function of the radius of relative stiffness. Figure 3 shows critical load positions.

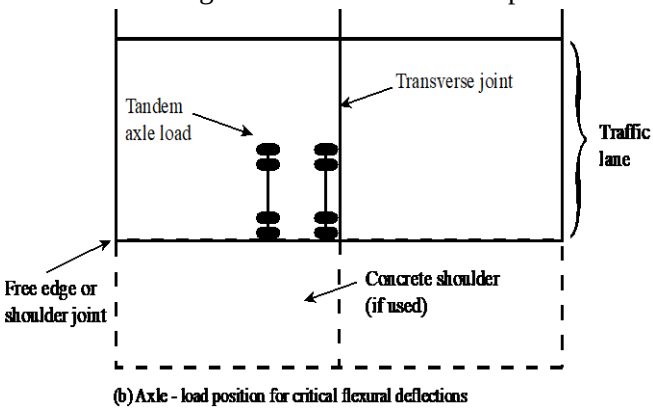

Figure 3: Critical loading position for erosion analysis
Figure 3 : Position de chargement critique pour l'analyse de l'érosion

The equation for the erosion:

$$\begin{cases} \log(N_e) = -\log(C_2) + 14.524 - 6.777(C_1P - 9)^{0.103} C_1P > 9 \\ N_e = Unlimited C_1P \leq 9 \end{cases} \quad (4)$$

$C_1=1-((k/2000)(4/h_1))^2$

$C_2=0.06$ for NS

$C_2=0.94$ for WS

where

N_e, the allowable load repetitions based on erosion;
C_1 is an adjustment factor; which has a value close to 1.0 for untreated sub-bases and decreases to approximately 0.90 for stabilized sub-bases; C_2, the adjustment factor for slab edge effects, 0.06 for base with no shoulder, 0.94 for base with shoulder;

The principal mode of failure was pumping or erosion of the granular sub-base. Thus the PCA's erosion analysis concept is to avoid pavement failures dues to pumping, erosion of foundation, and joint faulting, which are closely related to pavement deflection. The most critical pavement deflection occurs at the slab corner when an axle load is placed at the joint near to the corner.

The equivalent corner deflection δ_{eq}equations were developed for slabs with aggregate interlock joints or doweled joints under a single axle load or a tandem axle load:
PCA Equivalent corner deflection:

$$\delta_{eq} = \left(\frac{p_c}{k}\right) f_5 f_6 f_7 (5)$$

Single Axle (SA)/No Shoulder (NS)/No Doweled (ND)
$p_c= 1.571+(46.127/L) +(4372.7/L^2) –22886/L^3)$ SA/NS/ND
Tandem Axle (TA)/No Shoulder (NS)/No Doweled (ND)
$p_c= 1.847+(213.68/L) -(1260.8/L^2)+(22989/L^3)$ TA/NS/ND
Single Axle (SA)/With Shoulder (WS)/No Doweled (ND)
$p_c= 0.5874+(65.108/L) +(1130.9/L^2) – (5245.8/L^3)$ SA/WS/ND
Tandem Axle (TA)/With Shoulder (WS)/No Doweled (ND)
$p_c= 1.47+(102.2/L) -(1072/L^2) +(14451/L^3)$ TA/WS/ND
Single Axle (SA)/With Shoulder (WS)/ WithDoweled (WD)
$p_c= -0.3019+(128.85/L) +(1105.8/L^2) +(3269.1/L^3)$ SA/NS/WD
Tandem Axle (TA)/With Shoulder (WS)/With Doweled (WD)
$p_c= 1.258+(97.491/L) +(1484.1/L^2)-(180/L^3)$ TA/NS/WD
Single Axle (SA)/With Shoulder (WS)/With Doweled (WD)
$p_c= 0.018+(72.99/L) +(323.1/L^2) +(1620/L^3)$ SA/WS/WD
Tandem Axle (TA)/With Shoulder (WS)/With Doweled (WD)
$p_c= 0.0345+(146.25/L) -2385.6/L^2)+(23848/L^3)$ TA/WS/WD
δ_{eq}: equivalent corner deflection

f_5 : adjustment factor for the effect of axle loads
f_6 : adjustment factor for a slab with no doweled joints and no tied concrete shoulder
f_7 : adjustment factor to account for the effect of truck placement on the corner deflection

f_5= SAL/18 SA
f_5=TAL/36 TA

f_6 = 0.95 ND/NS
f_6 = 1.001-(0.26363-k/3034.5)^{2}ND/WS
f_6 = 1 WD

f_7 = 0.896 NS
f_7 = 1 WS

A better correlation was obtained by relating the performance to the rate of work or power (P) which is defined as the product of corner deflection (δeq) and pressure at the slab-foundation interface (p_c) divided by a measure of the length of deflection basin or the radius of relative stiffness.

$$P = \frac{268.7\left(k^{1.27}\delta_{eq}{}^{2}\right)}{h_1} \quad (6)$$

The determination of the well-known erosion factor in the PCA thickness design procedure was defined by:

ErosionFactor $(EF) = \frac{\log(11111\cdot(0896P)^2\cdot C_1)}{h_1\cdot k^{0.73}}$ (7)

CumulativeErosion Damage:

$$D = \sum_{i=1}^{m}\left(\frac{n_i}{N_{ie}}\right) \quad (8)$$

It is noted that the above equations are only applicable to US customary system (Imperial Units). Until proper adjustments to the coefficients in the equations, it cannot be directly used with pertinent input variables in metric unit (SI system) (Note the metric conversion factors:
1 in.=25.4mm; 1 lb=4.44822N;
1 psi=6.89476kPa; 1 psi/in. =0.27145MN/m^3).

Notation:
The following symbols are used:
Single Axle: SA
Single Axle Load: SAL
Tandem Axle: TA
Tandem Axle Load: TAL
No Shoulder: NS
With Shoulder: WS
No Doweled: ND
With Doweled: WD
Average Design Traffic Trucks: ADTT
Average Daily Traffic: ADT
Single Axle with Single Tyres: SAST
Single Axle with Dual Tyres: SADT
Tandem Axle with Single Tyres: TAST
Tandem Axle with Dual Tyres:TADT

3-Rigid pavement optimization problem

3.1 Design variables

The design variables selected for the optimization are presented in Table 1 and Fig. 1

Table 1: Definition of design variables

Tableau 1 : Définition des variables de conception

Design variables	Defined variables
h_1	Thickness of the concrete slab
h_2	Thickness of prepared subgrade

3.2 Cost function

The objective function to be minimized in the optimization problems is the total cost of construction material of the rigid pavement. This function can be defined as:

$$C_T = C_c h_1 \cdot b \cdot 1 + C_f h_1(2+b) + C_{sub} h_2 \cdot b \cdot 1 \quad (9)$$

C_T is the total cost of the rigid pavement per meter of the length,

C_c is the cost of the concrete slab per meter of the length,

C_f is the cost of the framework per meter of the length,

h_1 is the thickness of the concrete slab,

$C_{sub}$is the cost of the prepared subgrade per meter of the length,

h_2 is the thickness of the prepared subgrade,

It should be noted that in a cost optimization problem, the optimal values of the design variables are only affected by the relative cost values of the objective function and not by the absolute cost values. In other words, the absolute cost values affect the final value of the objective function but not the optimal values of the design variables.
The absolute cost C_T can then be recovered from the optimized relative cost C by using the relation:$C=C_T/C_c$
Thus, the objective function to be minimized can be written as follows:

$$C = h_1 \cdot b \cdot 1 + \left(\frac{C_f}{C_c}\right) h_1(2+b) + \left(\frac{C_{sub}}{C_c}\right) h_2 \cdot b.1 \quad (10)$$

3.3. Design constraints (structural capacity):

a) Minimum and maximum thickness constraints:

$$h_{1\min} \leq h_1 \leq h_{1\max} (11)$$

$$h_{2\min} \leq h_2 \leq h_{2\max} \ (12)$$

b) Non-negativity constraint:

$$h_1,\ h_2 \geq 0 \qquad (13)$$

c) Cumulative damage concept for fatigue analysisconstraint:

$$0 \leq Cumulative\ Fatigue\ Damage\ = \sum_{i=1}^{m} \left(\frac{n_i}{N_{if}}\right) \leq 1 (14)$$

d) Cumulative damage concept for erosion analysis constraint:

$$0 \leq Cumulative\ Erosion\ Damage\ = \sum_{i=1}^{m} \left(\frac{n_i}{N_{ie}}\right) \leq 1\ (15)$$

3.4 Optimization based on minimum cost design of concrete pavements

The optimum cost design of unreinforcedconcrete slabs can be stated as follows:

For given material properties, loading data and constant parameters, find the design variables defined in Table (1) that minimize the cost function defined in Eq. (10) subjected to the design constraints given in Eq. (11) through Eq.(15).

3.5 Solution methodology: Generalized Reduced Gradient method

The objective function Eq. (10) and the constraints equations, Eq.(11) through Eq.(15), together form a nonlinear optimization problem. In order to solve this nonlinear optimization problem, the generalized reduced gradient (GRG) algorithm is used. GRG non-linear should be selected if any of the equations involving decision variables or constraints is nonlinear.

The Generalized Reduced Gradient method is applied as it has the following advantages:

i) The GRG method is widely recognized as an efficient method for solving a relatively wide class of nonlinear optimization problems.

ii) The program can handle up to 200 constraints, which is suitable for rigid pavements design optimization problems.

iii) GRG transforms inequality constraints into equality constraints by introducing slack variables. Hence all the constraints are of equality form.

4. Numerical results and discussion

4.1 Design example

Pavement materials Data:
Design compressive strength of concrete; f'_c=35 MPa
Concrete modulus of rupture;
$f'_{cf}=0.75(35)^{0.5}$=4.44MPa
Modulus of subgrade-subbase reaction;
k = 81.434MPa/m
Modulus of elasticity of concrete;
E= 35000MPa=35GPa
Poisson's ratio of concrete: υ=0.15
Unreinforced concrete pavement with no concrete shoulder (NS) and no doweled joints (ND)
Two lanes in one direction with a total width b=7m
Slab length: 4m; slab width: 3.5m (per lane)
The thickness of the prepared subgrade is the same for both the conventional solution and the optimal solution ($h_{2classical}$ =$h2_{optimal}$ =300mm)
Input data for unit costs ratios of construction materials:C_{sub}/C_c= 0.10,C_f/C_c= 0.01
Traffics Data:
Load Safety Factor for axle loads LSF= 1.2
Axle load: 10KN ≤P≤ 400 KN
Average Design Traffic Trucks:
ADTT=365(ADT)(T)(D)(LDF)(G)(N)
Pavement Design Life (design period): N=20
Average Daily Traffic:
ADT=58267 vehicles /day
Direction Distribution Factor: D=0.50
Lane distribution factor (02 lanes in one direction): LDF=0.81
Percent trucks: T=20%
Annual traffic growth rate: r=4%
Growth factor: G=1.48

Total number of trucks on the design lane during the design period:

Average Design Traffic Trucks:
ADTT=365(ADT)(T)(D)(LDF)(G)(N)
=365(58267) (0.20) (0.50) (0.81) (1.48) (20)
ADTT =51000000.

Axle load distributions project by category for two lanes in one direction:

ADTT-SAST=22503734Load category one: Single Axle with Single Tyres (SAST)
ADTT-SADT=13008638Load category two: Single Axle with Dual Tyres (SADT)
ADTT-TAST=962852Load category three: Tandem Axle with Single Tyres (TAST)
ADTT-TADT=14524776Load category four: Tandem Axle with Dual Tyres (TADT)

Percentage of proportion by category:
ADTT-SAST (44,12%); ADTT-SADT (25,51%); ADTT-TAST (1,89%); ADTT-TADT (28,48%)

4.2 Comparison between the classical solution and optimal solution

The optimal solution using the minimum cost design is shown in Table 2 below.

Table 2: Comparison of the classical solution with the optimal solution
Tableau 2 : Comparaison de la solution classique avec la solution optimale

Design Variables Vector	Initial Design (Classical Solution)	Optimal solution
h_1 (mm)	292	260
h_2(mm) imposed	300	300
Cost $C=C_T/C_c$	92.165	83.208
Gain (%)	/	11
Cumulative Fatigue damage (%)	00	1.74
Cumulative Erosion damage (%)	36.68	100

From the above results, it is clearly shown that a significant cost saving of the order of 11 % through the use of minimum cost design approach.

5. Conclusions

The following important conclusions are drawn on the basis of this research:

- The problem formulation of the optimal cost design of concrete pavements can be cast into a nonlinear programming problem, the numerical solution is efficiently determined using the Generalized Reduced Gradient method.
- The optimal value of the design variable is only affected by the relative cost values of the objective function and not by the absolute cost values.

- The observations of the optimal solution result reveal that the use of the optimization based on the optimum cost design concept may lead to substantial savings in the amount of the construction materials to be used in comparison to classical design solutions of rigid pavements.
- The objective function and the constraints considered in this paper are illustrative in nature. This approach based on nonlinear mathematical programming can be easily extended to other cases commonly used in structural design. More sophisticated objectives and considerations can be readily accommodated by suitable modifications of the optimal cost design model.
- In this work, the additional cost of formwork is included which makes a significant contribution to the total costs.
- The suggested methodology for optimum cost design is effective and more economical comparing to the classical methods. The results of the analysis show that the optimization process presented herein is effective and its application appears feasible.

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